Using Relational Thinking Strategies to Establish the Relationship between Conceptual and Procedural Knowledge in Mathematics Problem Solving Classrooms

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***Abstract**: This study was conducted in a mathematics classroom context using the Lesson Study and Open Approach as the problem solving method. The purpose of this study was to investigate the students' usage of relational thinking strategies for establishing the relationship between conceptual and procedural knowledge. The researcher explained to the students the usage of relational thinking strategies to establish the conceptual and procedural knowledge by teaching experiment methodology. The target group included 6 fourth grade students in the academic year 2012 at Sa-Nam-Bin School, Khon Kaen Province. The data were collected by using the participatory observation, informal discussion, and in-depth interview with target group students. The research findings revealed that the relational thinking strategies for establishing the relationship between students' conceptual and procedural knowledge was to recognize the relationship between the corresponding number, the ability to construct the new number sentence with equivalence with former number sentences by writing the number of representation, and the ability to use the number sentence with equivalence in order to find the answer leading to a secure procedure and meaning type idea causing the occurrence of relationship between conceptual and procedural knowledge.*

Keywords: Mathematics Problem Solving Classroom, Relational Thinking Strategies, Relationship between Conceptual and Procedural Knowledge

Introduction

In the reform of mathematics learning in a mathematics classroom in Thailand, changing the teachers' teaching as well as students' learning is crucial. As suggested by Loipha and Inprasitha (2004), 'lesson study' is an innovation including the most efficient basic approach in improving as well as developing the classroom since 2002 by Dr. Maitree Inprasitha, a lecturer of the Faculty of Education, Khon Kaen University. The process of lesson study consists of three major steps: 1) collaboratively designing research lesson (Plan), 2) collaboratively observing research lesson (Do), and 3) collaboratively reflecting in teaching practice (See). These steps enable the cycle of lesson study to be continuously implemented in the context of Thailand. Inprasitha (2006) proposed that the 'open approach' was to integrate with the Lesson Study as well as innovation being able to improve mathematical teaching and learning very well.

The Mathematics classroom provides the learning by using the lesson study and open approach as a problem solving classroom. This type of classroom is based on an open-ended problem for causing the learning process through an open approach as a teaching approach including four steps: 1) Posing open-ended problems, 2) students' self-learning, 3) whole class discussion and comparison, and 4) summarization through connecting students' mathematical ideas which emerged in the classroom

(Inprasitha, 2010). The process of lesson study plays an important role in changing the teaching pattern from the traditional one to a student-centered teaching. The established lesson plan will lead to the open-ended problem giving an importance to the students' learning for understanding rather than teaching to cover the existing content in the curriculum. The change as well as transferring the teaching pattern in aligned with lesson study which makes the students learn mathematics meaningfully based on their previously learned knowledge leading them to develop new knowledge. This rationale concerning a problem solving classroom will enable the students to see the relationship between their ideas used in previously learned knowledge and the new ideas, and enable them to be able to construct the body of knowledge by themselves.

According to Kilpattrick, Swafford and Findell (2001), conceptual and procedural knowledge are the two of five strands of mathematical proficiency. Conceptual knowledge refers to the comprehension of mathematical concept, operation, and relations. Procedural knowledge, on the other hand, refers to the skills in carrying out procedures flexibly, accurately, efficiently, and appropriately (Kilpattrick et al, 2001). The relationship between conceptual and procedural knowledge is an important objective of mathematics instructional management in primary education. Seeing the relationship emphasizes the relational understanding as meaningful understanding by focusing on the explanation by giving an example in each concept as well as procedure and being able to form the association between the mathematics concept and procedure (Loipha, 2003). Furthermore, in mathematics learning, the students would learn the mathematics concept and procedure by extending this from prior learning. To be precise, the students are likely to learn the mathematics concept and procedure by associating the relationship from prior knowledge with the obtained new knowledge (Hiebert & Carpenter, 1992). However, most students cannot see the relationship between the two types of knowledge and think that there is no relationship between them.

Relational thinking is a process to support the students' learning to see the relationship between different mathematical ideas, concepts, and procedures. The establishment of a precise relationship through relational thinking will be a specification of the relationship as the core of understanding mathematics more clearly (Koechler, 2004). The relational thinking also play its role in the foundation of arithmetic learning as a supporter for seeing the arithmetic more than only computation for solution only (Carpenter, Franke & Levi, 2003). Several mathematics educators including Stephens et al. (2007) and Carpenter et al. (2003) give the example of the number sentences 47 + 25 which can be transformed into 50 + 22 by "adding 3" to 47 and "subtracting 3" from 25. They claim that when students apply this strategy to sensibly solve different numerical problems, they disclose an understanding of the relationships of the number involved. These educators refer to the thinking underpinning this kind of strategy as relational thinking.

The systematic strategies lead to the enhancement of the relationship between conceptual and procedural knowledge needed to be a strategy for reducing the students' usage of overgeneralized ideas while they are solving mathematics problems (Isoda, 1996). The overgeneralized ideas would apply procedural knowledge learned from a topic of content in solving other topics by previously learned tasks until forgetting the conceptual knowledge in such a topic. As a result, this leads to the gap between the two types of knowledge, and thus no relationship between conceptual and procedural knowledge is seen to exist (Isoda, 1996). The current research problem deals with how the relationship between conceptual and procedural knowledge occurs. Previous studies (e.g. Rittleston-Johnson & Siegler (1998); National Research Council (2001); and Siegler & Alibali (2001)) dealing with the characteristics in establishing the relationship between conceptual and procedural knowledge used a test as an instrument and analyzed it quantitatively, this resulted in the limitation in analyzing the students' overgeneralized ideas. Consequently, they were not able to see the details of ideas, widening the gap between conceptual and procedural knowledge. To solve this methodological problem, the present study used relational thinking strategies to bridge the gap between the two types of knowledge.

Isoda (1996; p: 226) specified "mechanism of classroom learning to the decrease of gaps between conceptual and procedural knowledge" as the framework for analyzing the students' various ideas caused from problem solving in a problem situation and led to the elimination of the gaps between conceptual and procedural knowledge which could be used in analyzing a variety of ideas causing the classroom discussion. Moreover, Isoda (2013; p: 147) determined the "mechanism of sub-unit construction which caused variety of ideas" as the framework for analyzing the sub-learning unit in the level of the lesson plan. According to the studies of mechanism in classroom leading to the elimination of the gaps between conceptual and procedural knowledge, Isoda (1996, 2013), for example, found that the students used five types of problem solving: 1) Prioritize procedure without meaning type, 2) Prioritize procedure with confused or ambiguous meaning type, 3) Secure procedure and meaning type, 4) Prioritize meaning without procedure (or confused) type, and 5) No meaning and procedure. Type 3 approach would be the ideas causing the relationship between conceptual and procedural knowledge. This was because the students realized the relationship between the ideas of their previously learned tasks and the target tasks in order to construct the new knowledge.

Therefore, in the present study, Isoda's approach was used as a basis in constructing the framework used as a guideline for analyzing the students' ideas reflecting the relationship between prior knowledge and the new knowledge which would be able to understand the details of students' ideas and utilize these in developing their mathematics learning further.

The relational thinking strategies are systematic strategies which would lead to the occurrence of an idea using the correct concept as well as procedure. Since the students express their ability in seeing the overall number sentence as well as the relationship between number and the implementation of a related number occurring in number sentence and obtaining the answer without using only arithmetic computation step-by-step. This procedure is related to the understanding in equal sign, compensation to change the pattern of number sentence into an easy form in order to find the answer, and the use of equivalence reducing the occurring gaps. As a result, the relationship between conceptual and procedural knowledge occurs.

Using the relational thinking strategies to establish the relationship between conceptual and procedural knowledge is the study of strategies where the students express their ability in relational thinking through the usage of compensation and equivalence. These strategies cause the students to obtain ideas in using a secure procedure and meaning type using the establishment of the relationship between conceptual and procedural knowledge. Using relational thinking strategies should include the consideration of the mechanism of relational thinking strategies in order to construct the relationship between conceptual and procedural knowledge in the problem solving classroom process as well as developing a framework constructed by the researcher as a framework for data analysis.

Therefore, this study is developed under the context of a lesson study and using an open approach as a problem solving classroom. This is done by using the real problem situation causing the students to express their thinking strategies focusing on them to see the relationship of knowledge they use to obtain meaningful association to the occurrence of new ideas. Thus, the study for viewing the guidelines in establishing the relationship between conceptual and procedural knowledge should be studied in the context that enables the students to view or learn how to alleviate difficulty during problem solving, leading to the discovery of a law or principle to be used with solving new problems. From the discussion, it can be seen that, the establishment of the relationship based on the implementation of relational thinking strategies in the mathematics classroom context using lesson study and an open approach is an efficient guideline in developing meaningful mathematics learning.

Research Objective

The objective of the research is to investigate the relational thinking strategies to establish the relationship between conceptual and procedural knowledge in a mathematics problem solving classroom.

Research Design

This study is conducted by synthesizing the documents as well as related research literature and empirical data in classrooms in order to support the research hypothesis. The use of relational thinking strategies between conceptual and procedural knowledge is analyzed by using the teaching experiment methodology. The data are collected by means of participatory observation, informal discussion, and in-depth interviews with the target group students.

The students participating in this research were 38 Pratom Suksa 4 students, Sa-nam-bin School, Muang District, Khon Kaen Province. The school was selected because it was under the Teaching Profession Development Project and used the lesson study and an open approach. All the students in Pratom Suksa 4 class (n=38) were administered with the Relational Thinking Test developed by Stephens, Isoda and Inprasitha (2007) to select the sample group who used relational thinking strategies in finding the solutions to the test items.

Upon completion of the Relational Thinking Test, fourteen students were found to use relational thinking strategies. Apart from using the strategies, the present study also aimed to select the students who could elaborate their thinking into words. Therefore, in addition to the test, in-depth interviews concerning the conceptual and procedural knowledge and the association between the two kinds of knowledge based on relational thinking were also employed. After the selection process, the sample group of six students was chosen—one student from each level (1-4) and two students expressing the relational thinking behavior in their performances, the responses from the informal interviews or protocol during their problem solving who did not express their relational thinking ability in responding to the test.

Data Collection and Analysis

This study centered on the engagement in the learning activities in the five problem situations as target tasks on the rule of division. To collect the data on the students' thinking and the usage of relational thinking strategies, the occurrences in classroom in the learning step were videotaped, and the analysis also concerned field notes, the teachers and the observers' interviews and students' performance analysis for explaining as well as interpreting the students' speaking and action. The data were analyzed using the framework of analysis in relational thinking strategies constructed by the researcher based on Isoda (1996), Inprasitha (2010) and Stephens et al. (2007) approaches.

The Framework of Analysis in Relational Thinking Strategies

The framework of analysis in relational thinking strategies emphasizing the occurrence of the relationship between conceptual and procedural knowledge and the working mechanism in relational thinking strategies in mathematics problem solving classroom is shown in Figure 1.

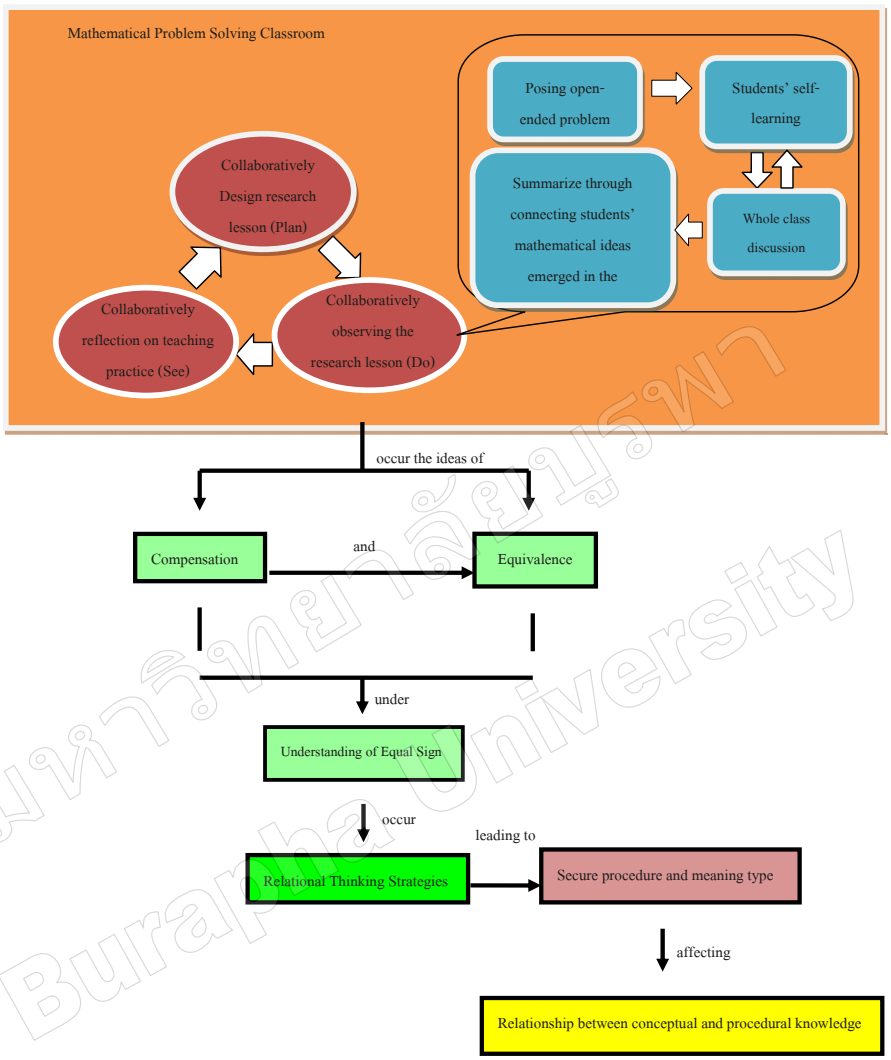

Figure 1: Framework of Analysis in Relational Thinking Strategies based on Inprasitha (2010), Stephens et al. (2007), and Isoda (1996)

Research Results

The study of the usage of relational thinking strategies to establish the relationship between conceptual and procedural knowledge is discussed in three major issues as follows:

1. The composition of relational thinking strategies consisted of the comprehension in the equal sign, the usage of compensation and equivalence. Considering the composition of relational thinking strategies occurring in a mathematics problem solving classroom, it was found that the students were able to explain that the equal sign was the symbol showing the relationship between the two equal quantities, and the number to be added in the sentence had to be a number being able to make two sides of the equal sign, to have equal value. The students also expressed their ability in understanding the equal sign from:

1) *The usage of compensation:* The students were able to use the process in changing the form of number sentence based on filling parts in order to be find the solution more easily.

2) *The usage of equivalence:* The students were able to explain the reasons why the numbers through the process of compensation were the equivalence sentences.

2. The relationship between conceptual and procedural knowledge occurred when the students employed the relational strategies leading to the secure procedure and meaning type idea. As a result, the students could see associated thinking approaches or ideas for problem solving in previously learned tasks and the ideas for solving the target tasks.

3. The mechanism of working in relational thinking strategies for establishing the relationship between conceptual and procedural knowledge was the process causing students to obtain their problem solving ideas by using secure procedure and meaning type idea which resulted in the decrease of the gaps between conceptual and procedural knowledge. The findings reveal that the students were able to write number representation variously, including the clue in writing the associated numbers, the use of arrow, or explanation for comparing the number, the use of comparison in number to help in searching for the answer. Finally, when the students used the number sentence, the new sentence being equivalence with that sentence can be obtained to find the same correct answer.

To summarize, from the three analyses of 1) the composition analysis and the details of each composition in relational thinking strategies, 2) the occurrence in the relationship between conceptual and procedural knowledge, and 3) the mechanism of such strategies in mathematics problem solving classroom, it was found that the usage of relational thinking strategies in developing the relationship between conceptual and procedural knowledge were to: 1) see the relationship between the corresponding number, 2) use the number sentence being equivalence with the former sentence based on the writing for number representation, and 3) use the number sentence with equivalence in searching for the answer leading to the ideas using secure procedure and meaning type idea affecting the relationship between conceptual and procedural knowledge.

Additionally, the existence of the usage of relational thinking strategies for developing conceptual and procedural knowledge can be supported by the students' thinking in classroom observation on the problem situation on "The target task number 1" on the Four Phases of open approach as the teaching approach which is displayed in Figure 2.

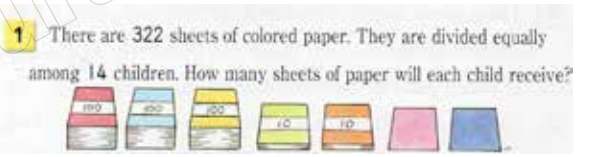

Figure 2: Problem situation for the students to find the solution

Phase 1: Posing open-ended problem: The objective of this task was to enable the students to find the answer by the rule of division.

Phases 2 and 3: students' self-learning and whole class discussion and comparison: First, the students brought 14 to divide both of the divided and the divisor to make the new number sentence, as shown in Figure 3.

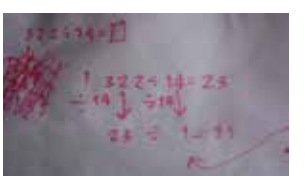

Figure 3: The students' idea for making the new number sentence

The above figure shows that the students tried to use the idea of the rule of division as learnt before, but they misconceived about the selection of the divisor. The gaps between conceptual and procedural knowledge had occurred. When the students participated in the discussion, they had to compare their way of thinking with that of others and discuss their process finding with other students. In this sequential flow, relational thinking strategies in developing the relationship between conceptual and procedural knowledge occurred, as shown in Figure 4.

Figure 4: Students' Relational Thinking Strategies

Phase 4: Summarization by connecting the students' mathematical ideas emerging in the classroom. At this point, the teacher and the whole class collectively concluded the lesson learnt.

Conclusion and Discussions

The usage of relational thinking strategies in developing the relationship between conceptual and procedural knowledge occurred during the implementation of problem solving. The students employed the relational thinking strategies in order to see the relationship between conceptual and procedural knowledge occurring during Step 2 or Step 3 of open approach. The findings revealed that in the context of lesson study and open approach as the problem solving classroom, the relationship between conceptual and procedural knowledge would occur when the students obtained ideas of "Secure procedure and meaning type."

Seeing the associated approaches between the ideas of students' previously learned tasks and target tasks is based on systematic thinking which supports the students to learn how to see the relationship between mathematical thinking, concepts, and procedure. This finding is in line with Koechler's (2004) study which found that the development of precise association through relational thinking was the specification of the relationship as the core of comprehension in mathematics. In addition, the systematic strategies leading to the enhancement of the relationship between conceptual and procedural knowledge had to take part in reducing the students' overgeneralized idea in solving mathematics problems. The overgeneralized idea was to use procedural knowledge in one topic of content for solving the problem in other topics by previously learned tasks until they forgot to think about conceptual knowledge of those topics. As a result, there were the gaps between the two kinds of knowledge leading to no relation or association between them (Isoda, 1996).

The overall findings of this study suggest that when the students are faced with a problem situation and express the other ideas in problem solving, relational thinking strategies are guidelines for the students to discover the idea called "Secure procedure and meaning type" through classroom interactions either between student-student, or teacher-student.

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